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# RESPONSE CHARACTERISTICS OF MAGNETIC AMPLIFIERS WITH EXTERNAL FEEDBACK

BY

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by

Richard Earl Seacord
Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
**in**
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1951

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# CONTENTS

Page

List of Illustrations........................... iii

Table of Symbols and Abbreviations............... iv

Chapter I - Introduction......................... 1

Chapter II - Analysis............................ 10

Chapter III - Conclusions........................ 29

Bibliography..................................... 31

Appendix......................................... 33

## ILLUSTRATIONS

Page

Figure 1 - Series connected magnetic amplifier with external feedback.................... 5

Figure 2 - B-H characteristic assumed in analysis... 11

Figure 3 - Parallel connected magnetic amplifier with external feedback.................... 12

Figure 4 - Currents in main winding branches for parallel connection.................. 13

Figure 5 - Series connection...................... 14

# SYMBOLS AND ABBREVIATIONS

$B$ - magnetic flux density in core

$e_1$ - alternating voltage driving magnetic amplifier

$e_3$ - voltage at signal winding terminals

$E_3$ - magnitude of step change in $e_3$

$H$ - magneto-motive force

$i_A$ - instantaneous current in main winding of core A, parallel connection

$i_B$ - instantaneous current in main winding of core B, parallel connection

$i_L$ - instantaneous current through load

$i_1$ - instantaneous current in main windings in series connection

$i_3$ - current in signal winding, all connections

$K$ - inductance coefficient including core dimensions

$N_1$ - number of turns per core on main windings

$N_2$ - number of turns per core on feedback winding

$N_3$ - number of turns per core on signal winding

$q$ - operator $K\mu S$

$S$ - Laplace operator

$\alpha$ - ratio of feedback ampere-turns to load ampere-turns

$\phi$ - instantaneous value of flux in core

$\psi$ - "firing" angle, electrical angle after beginning of cycle at which saturation level of core is reached

$\mu$ - permeability determined by slope of B-H characteristics prior to saturation

# CHAPTER I

# INTRODUCTION

The advent of magnetic amplifiers into the field of servo mechanisms has served to emphasize a major disadvantage of these devices, their slow speed of response. At present all magnetic amplifiers are current controlled devices, making the response problem basically that of changing the current in an inductive-resistive signal circuit. Few generalities can be made about the characteristics of these devices since individual amplifiers vary widely in types of core materials and circuit arrangements. The type of load will affect the operation of any given amplifier. For these reasons specific circuit arrangements have been chosen for study.

Any analytical determination of the characteristics of non-linear devices can be at best approximate. Validity of the results will depend upon the variation of the actual behavior of material from that assumed in the analysis. Progress in the development of saturable reactor core materials has brought the actual behavior of these materials into close conformity with that assumed in several mathematical analyses (3, 6, 7, 9). The assumptions made on the behavior of core material will determine the type of analysis to a large extent. Another consideration in choosing an analytical approach is the form in which the results are desired. For example, the response characterisitcs may be expressed as response time, that is the time elapsing between the application of a step voltage at the input terminals and the attainment of a new average level of flux density, or by an expression for the

instantaneous value of signal current. Up to the point of introduction of the assumed B-H relationships, the types of attack to the problem are similar in that voltage equations around the several loops of the circuit are written in terms of the resistances, number of turns on individual windings and time rates of change of flux in these windings. The simplified analyses ignore the effects of leakage flux and hysteresis as well as the variable forward resistances of rectifiers where employed.

A group at Carnegie Institute of Technology including D. W. Ver Planck, M. Fishman, D. C. Beaumarriage and L. A. Finzi (7, 8, 9) have investigated transients in magnetic amplifiers by assuming a hyperbolic sine function to represent the B-H characteristics of certain core materials. From the relationships

$$\phi = \frac{10^8}{N}\int e\,dt \qquad e = E_m \cos \omega t$$

such that $$\phi = \frac{10^8}{\omega N} \sin \omega t + \phi_o$$

or $$B = B_m \sin \omega t + B_o$$

and $$H = U \sinh uB + vB$$

This group has developed an expression for the time required to change the direct component of flux density from an initial value $B_{oo}$ to another value $B_{ot}$ of the form

$$t = \frac{A}{Uu\ell} \sum_v \frac{N_v^2}{R_v} \int_{B_{oo}}^{B_{ot}} \frac{B_{ot}}{\frac{1}{\ell u}\sum_v N_v I_{v\infty} + \frac{2m}{\pi}\frac{N_f}{N_2}\cosh B_{ot} - n \sinh B_{ot}}$$

for the series connected amplifier with external feedback where

$A$ = cross section area of core

$U, u,$ are constants referring to the B-H characteristics

$\ell$ = length of magnetic circuit

$N_v$ = number of control turns and bias turns on separate windings

$N_f$ = number of feedback turns

$N_2$ = number of turns on main winding

$I_{v\infty}$ = final steady state value of control or bias current

$m, n$ = coefficients obtained from the expansion of the B-H relationship in modified Bessel functions.

It is indicated that a similar analysis can be made on the parallel reactor circuit. While this type of analysis will predict the time involved in a change of output level, it does not point to the influence of the various circuit constants on the instantaneous value of signal current other than to show the increase in response time as $\sum \frac{N_v^2}{R_v}$ is increased. To demonstrate the direct influence of the $\frac{N^2}{R}$ ratios a different attack on the problem may be justified, even though the assumptions on the magnetic behavior of the material are not as realistic as those made above.

If the B-H curve is assumed to consist of two straight line portions intersecting at the knee, or saturation "point", the inductance parameters are then treated as linear over two separate regions and the instantaneous current expressions may be derived for the circuit conditions existing for any combination of current direction and state of core. This type of analysis has been well illustrated by Lamm (7) and Krabbe (6). Its basic disadvantages are its failure to describe operation over the curved portion of the B-H characteristic and the difficulty in fitting boundary values at the region limits. However, if the current disturbances caused by the assumed sudden change in value of parameters do not materially alter the currents in respect to those which would exist for a slower change in

parameter as described by more accurate B-H approximations, the response characteristics should be in reasonably close agreement.

A simplified analysis described by Reyner (9) of the series connection is based on an equality between signal ampere turns and load ampere turns and on the rate of change of net average flux over a complete cycle with respect to the change of load current. The source of this analysis is the work of Atkinson and Gale (1). The circuit described is the series arrangement shown in Figure 1, employing the following values:

$$R_1 = 0\ ,\ R_2 = 0\ ,\ R_3 = R_d\ ,\ N_1 = N_a\ ,\ N_2 = N_f\ ,\ N_3 = N_d$$

$$e_3 = E_d \quad \text{and} \quad e_1 = E_{a\,max} \sin \omega t$$

Basic assumptions are a rectangular B-H characteristic in which the incremental permeability is infinite up to the saturation level, then zero thereafter, and the equivalence

$$I_d N_d = I_a N_a (1-\alpha)$$

where $\alpha$ is the feedback ratio $\dfrac{I_f N_f}{I_a N_a}$

Letting $\bar{\phi}$ represent the mean value of flux, the voltage equation representing conditions in the signal circuit over a considerable number of cycles becomes

$$E_d = R_d I_d + \frac{N_d}{10^8} \frac{d}{dt} (\bar{\phi}_A - \bar{\phi}_B)$$

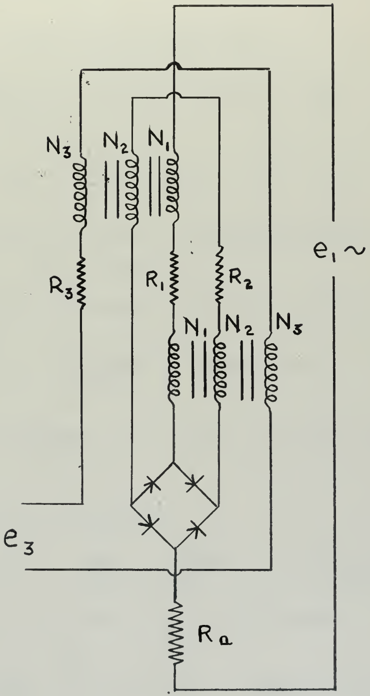
or, employing the equivalence of signal and load ampere-turns corrected for feedback,

$$E_d = \frac{N_a (1-\alpha) R_d}{N_d} I_a + \frac{N_d}{10^8} \frac{d}{d I_a} (\bar{\phi}_A - \bar{\phi}_B) \frac{d I_a}{dt}$$

During the "off" portion of the cycle, when both cores are in the unsaturated condition

Figure 1

Series Connection



$$\frac{d\phi_A}{dt} = \frac{d\phi_B}{dt} = \frac{10^8 E_{a\,max} \sin \omega t}{2\,N_a}$$

for a core A saturated: $\frac{d\phi_A}{dt} = 0$ , and for core B saturated: $\frac{d\phi_B}{dt} = 0$

so that over one complete cycle

$$\bar{\phi}_A + \bar{\phi}_B = 2\phi_{sat.}, \text{ or } \quad \bar{\phi}_A - \bar{\phi}_B = 2\left(\phi_{sat.} - \bar{\phi}_B\right)$$

where

$$\bar{\phi}_B = \int_0^{2\pi} \phi_B \; d\omega t$$

From the flux waveform resulting from the basic assumptions, it is found that $\bar{\phi}_B = \frac{1}{2}\phi_{B\,max}$.

If $\psi$ is the value of $\omega t$ at the instant of saturation of core B,

$$\phi_{B\,max} = \frac{10^8 E_{a\,max.}}{2\,N_a\,\omega} \int_0^{\psi} \sin \omega t \;\; d\omega t$$

$$\phi_{B\,max} = \frac{10^8 E_{a\,max.}}{2\,N_a\,\omega} \left(1 - \cos\psi\right)$$ and since $\phi_{sat.}$

is independent of time and firing angle

$$\frac{d}{d\psi}\left(\bar{\phi}_A - \bar{\phi}_B\right) = -2\frac{d\bar{\phi}_B}{d\psi} = -\frac{d\phi_{B\,max}}{d\psi} = -\frac{10^8 E_{a\,max} \sin\psi}{2\,N_a\,\omega}$$

the average value of output current, $I_a = 2f\int_0^{\pi} i_A \, d\omega t$

or $\quad I_a = 2f\,\frac{E_{a\,max}}{\omega\,R_a}\int_{\psi}^{\pi} \sin\omega t \; d\omega t$ because the output current is zero during the portion of the cycle $(0-\psi)$ in which the permeability is infinite.

then $\frac{d I_a}{d \psi} = - \frac{2f E_{a\,max} \sin \psi}{\omega R_a}$ and

$$\frac{d}{d I_a}(\bar{\phi}_A - \bar{\phi}_B) = \frac{d}{d\psi}(\bar{\phi}_A - \bar{\phi}_B)\frac{d\psi}{d I_a} = \frac{E_{a\,max} 10^8 \sin\psi}{2 N_a \omega} \; \frac{\omega R_a}{2f E_{a\,max} \sin\psi}$$

$$= \frac{R_a \, 10^8}{4f \, N_a}$$

Resulting in the following equation for $E_d$ :

$$E_d = \frac{N_a(1-\alpha)R_d}{N_d} I_a + \frac{R_a N_d}{4f \, N_a} \frac{d I_a}{dt}$$

yielding $I_a = \frac{E_d N_d}{R_d N_a (1-\alpha)} \left[1 - \epsilon^{-t/T}\right]$

where $T = \frac{R_a N_d^2}{R_d N_a^2 (1-\alpha) 4f}$

the power gain, $G = \frac{I_a^2 R_a}{I_d^2 R_d}$ and from the equivalence

expression, $G = \frac{N_d^2 R_a}{(1-\alpha)^2 N_a^2 R_d}$

so that $T = G(1-\alpha)/4f$

This time constant pointedly demonstrates the dependency of the normal response characteristics on amplifier gain. The presence of frequency in the denominator indicates that the time constant expressed in cycles of driving voltage is uniquely determined by the gain. It can be seen in review that the frequency is introduced by the fact that the flux

developed in either core during the "off" portions of a cycle depends inversely on the frequency and that the rate of change of net average flux with change in firing angle ($\psi$) retains this dependency, whereas the rate of change of average output current with respect to firing angle is independent of frequency.

The assignment of a precise time constant to any transient occurring over a given cycle in saturable cored circuits is somewhat misleading since the length of time during the cycle in which a core is saturated varies as the signal current is varied. Time constants may, however, be assigned to transient currents in certain zones of circuit behavior as defined by the degree of saturation of the cores and the action of the rectifier bridge, Krabbe (6) Chapter II. These time constants will serve to indicate the rate of change of the direct component of signal current in these zones. In the absence of any assumed or proved relationship between signal ampere turns and load ampere turns it may not remain clear that the output level, or average value of load current, depends on the value of the direct component of signal current. This fact is, however, the principle on which is based the use of d.c. controlled reactors in amplifiers. A determination of the instantaneous values of signal current and its direct component is a difficult matter when using any method. It is perhaps made more difficult by the assumption of a two-region straight line B-H characteristic, but it is felt that this approach makes the influence of the turns and resistances of the separate windings on the rates of change of signal current more apparent than other methods which may give more satisfactory quantitative results.

## Summary

To demonstrate the influence of certain factors on the rate of change of the direct component of signal current, hence, on the output level of external feedback magnetic amplifiers, a simplified analysis of two common circuit arrangements, namely, the parallel connection and the series connection, is made. The analysis is based on the assumption of a B-H characteristic for the material in which

$\frac{dB}{dH}$ is constant until the knee is reached, after which

$\frac{dB}{dH}$ is zero.

# CHAPTER II

## ANALYSIS

### 1. Assumptions

The theoretical B-H curve assumed for core material is shown in Figure 2.

Two separate cores are used in both the parallel and series arrangements in Figures 3 and 5. Cores are labelled A and B. Each core has $N_1$ turns in the load winding, $N_2$ turns in the feedback winding and $N_3$ turns in the signal winding. The cores and windings are symmetrical.

There is no leakage.

Hysteresis effects are negligible.

The rectifiers present no resistance to current in the forward direction and infinite resistance to reverse currents. Note in this respect that rectifier resistance to forward current could be included in the resistance of the branch following the rectifier. However, since this forward resistance is variable, it will be ignored.

The load is a pure resistance.

### 2. Parallel connection

A schematic diagram of the parallel connected, external feedback, magnetic amplifier is shown in Figure 2. The current $i_A$ is that flowing in the load winding $N_1$, on coil A, whereas $i_B$ is that flowing in the load winding $N_1$ on coil B. The term $R_T$ is the sum of $R_1$, $R_2$ and $R_L$. When both $i_A$ and $i_B$ are positive, the voltage equations are:

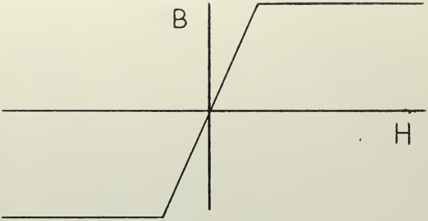


Figure 2

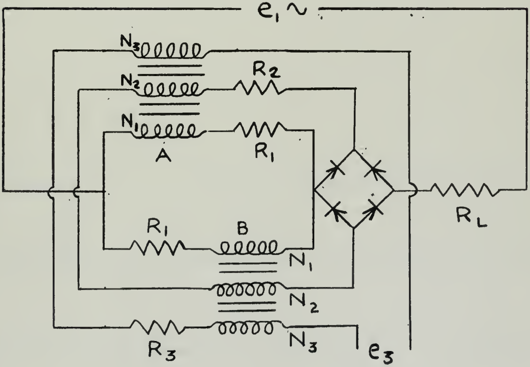


Figure 3

Parallel Connection

Figure 4

Main winding currents, parallel connection

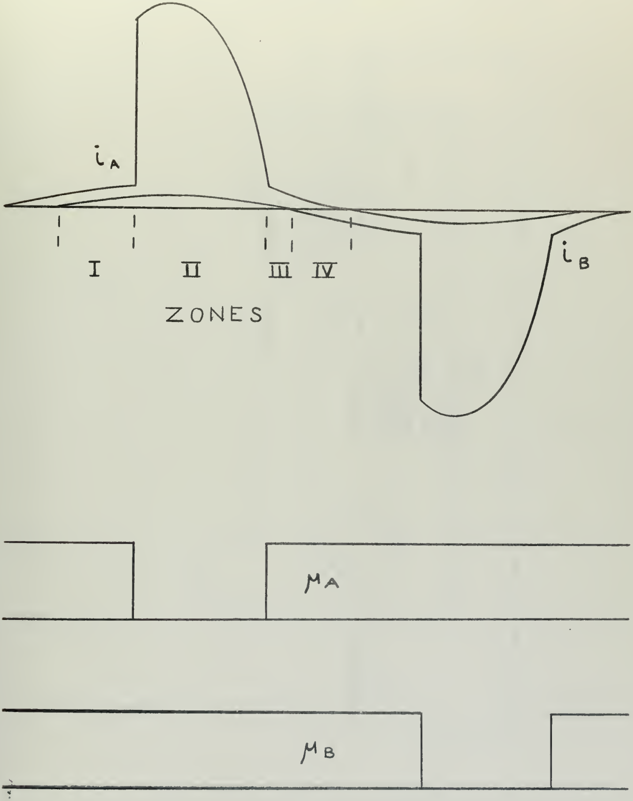
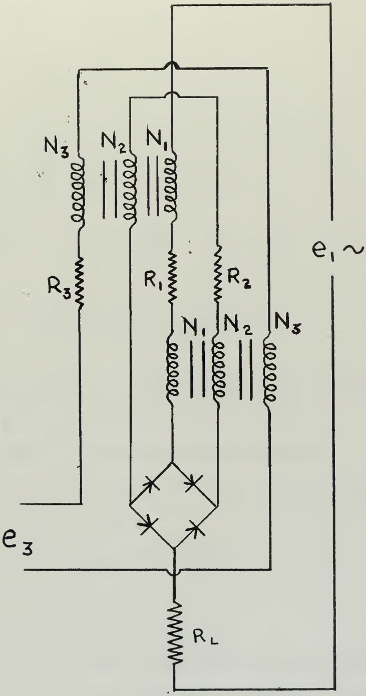


Figure 5

Series Connection



$$e_1 = i_A R_T + i_B (R_2 + R_L) + \frac{N_1}{10^8}\frac{d\phi_A}{dt} + \frac{N_2}{10^8}\frac{d\phi_A}{dt} + \frac{N_2}{10^8}\frac{d\phi_B}{dt}$$

$$e_1 = i_B R_T + i_A (R_2 + R_L) + \frac{N_1}{10^8}\frac{d\phi_B}{dt} + \frac{N_2}{10^8}\frac{d\phi_A}{dt} + \frac{N_2}{10^8}\frac{d\phi_B}{dt}$$

$$e_3 = i_3 R_3 + \frac{N_3}{10^8}\frac{d\phi_A}{dt} + \frac{N_3}{10^8}\frac{d\phi_B}{dt}$$

The terms $\frac{N_u}{10^8}\frac{d\phi}{dt}$ may be written

$$K N_u \sum_v \frac{d\phi_A}{di_v}\frac{di_v}{dt} \quad \text{and} \quad K N_u \sum_v \frac{d\phi_B}{di_v}\frac{di_v}{dt}$$

where $\frac{d\phi_A}{di_v} = N_v \mu_A$ and $\frac{d\phi_B}{di_v} = N_v \mu_B$

$$K = \frac{3.19}{10^8} \quad \frac{\text{area of core cross section}}{\text{magnetic length of core}}$$

From figure 1: $\mu_A = \mu_B = \mu$ before saturation of either core,

$\mu_A = 0$, $\mu_B = \mu$ after saturation of coreA

$\mu_B = 0$, $\mu_A = \mu$ after saturation of coreB.

The time region in which $l_A$ and $l_B$ are both positive during a given cycle may be divided into three zones:

Zone I: $\mu_A = \mu_B = \mu$, currents increasing

Zone II: $\mu_A = 0$, $\mu_B = \mu$

Zone III: $\mu_A = \mu_B = \mu$, currents decreasing

The subscripts $'$, $''$, $'''$ will be applied to currents and voltages existing immediately before the corresponding zone boundary is crossed. It may be seen that the voltage equations for Zone I and Zone III are identical except that $e_{1'} \neq e_{1'''}$ and $i_{v'} \neq i_{v'''}$ (except by chance). For

a step input in signal voltage $e_{3'} = e_{3'''}$ .

Manipulation of the equations will be simplified by the use of Laplace transforms, employing the notation

$$\mathcal{L}\,[f(t)] = \bar{f}$$

and adopting the identity $q_A = K \mu_A S$ where S is the Laplace operator.

The transformed voltage equations are

$$\begin{aligned}\bar{e}_1 + Y_A = {} & [q_A(N_1+N_2)^2 + q_B N_2^2 + R_T]\bar{i}_A + [q_A N_2(N_1+N_2) - q_B N_2(N_1-N_2) + R_2 + R_L]\bar{i}_B \\ & + [q_A N_3(N_1+N_2) + q_B N_2 N_3]\bar{i}_3\end{aligned}$$

$$\begin{aligned}\bar{e}_1 + Y_B = {} & [q_A N_2(N_1+N_2) - q_B N_2(N_1-N_2) + R_2 + R_L]\bar{i}_A + [q_A N_2^2 + q_B(N_1-N_2)^2 + R_T]\bar{i}_B \\ & + [q_A N_2 N_3 - q_B N_3(N_1-N_2)]\bar{i}_3\end{aligned}$$

$$\begin{aligned}\bar{e}_3 + Y_3 = {} & [q_A N_3(N_1+N_2) + q_B N_2 N_3]\bar{i}_A + [q_A N_2 N_3 - q_B N_3(N_1-N_2)]\bar{i}_B \\ & + [q_A N_3^2 + q_B N_3^2 + R_3]\bar{i}_3\end{aligned}$$

where

$$\begin{aligned}Y_A = {} & \frac{1}{S}[q_A(N_1+N_2)^2 + q_B N_2^2]i_{A'} + \frac{1}{S}[q_A N_2(N_1+N_2) - q_B N_2(N_1-N_2)]i_{B'} \\ & + \frac{1}{S}[q_A N_3(N_1+N_2) + q_B N_2 N_3]i_{3'}\end{aligned}$$

$$\begin{aligned}Y_B = {} & \frac{1}{S}[q_A N_2(N_1+N_2) - q_B N_2(N_1-N_2)]i_{A'} + \frac{1}{S}[q_A N_2^2 + q_B(N_1-N_2)^2]i_{B'} \\ & + \frac{1}{S}[q_A N_2 N_3 - q_B N_3(N_1-N_2)]i_{3'}\end{aligned}$$

$$\begin{aligned}Y_3 = {} & \frac{1}{S}[q_A N_3(N_1+N_2) + q_B N_2 N_3]i_{A'} + \frac{1}{S}[q_A N_2 N_3 - q_B N_3(N_1-N_2)]i_{B'} \\ & + \frac{1}{S}[q_A N_3^2 + q_B N_3^2]i_{3'}\end{aligned}$$

for Zone I and similarly for Zones II and III with appropriate subscript notation for initial currents.

The transform expressions for all currents in Zones I, II and III have been developed (Appendix I).

When both $i_A$ and $i_B$ are negative, the current equations are similar in form to those given here except that $i_B$ will behave as $i_A$ in the both positive zones and $i_A$ will behave as $i_B$ in these zones. The behavior of signal current, $i_S$, will be unchanged. In the intermediate, or commutation, zone one of the primary currents becomes negative while the other is still positive and a new set of voltage equations is required. These will be developed later for Zone IV. At the beginning of commutation after the both positive zones, $i_{B^{iv}}$ will be zero. At the beginning of Zone I for the both positive condition $i_{A'}$ will have some positive value other than zero, whereas $i_{B'}$ will be zero.

As noted in the introduction, the output level of the amplifier will change as the direct component of signal current changes such that the response characteristics of the amplifier may be determined by the time variation of the direct component of signal current in response to the voltage impressed on the signal winding terminals. For example, if, in response to a step input in signal voltage, the direct component of signal current reaches 90% of its final value in t seconds, the average value of load current will be at or above 90% of its final value less than one-half cycle of load current later (that is, over the next and successive "on" periods).

Letting $R_T = R_1 + R_2 + R_L$

and $R_P = R_T + R_2 + R_L$

in Zones I and III:

$$\bar{i}_3 = \frac{1}{R_3}\left\{ \frac{1}{q^2\left(\frac{N_1^4}{R_1 R_P}+\frac{2N_1^2N_3^2}{R_P R_3}\right)+q\left(\frac{2N_1^2}{R_1\left(1+\frac{R_2+R_L}{R_T}\right)}+\frac{4N_2^2}{R_P}+\frac{2N_3^2}{R_3}\right)+1}\right\}$$

$$\left\{-\bar{e}_1\frac{4qN_2N_3}{R_P}+\bar{e}_3\left(\frac{q^2N_1^4}{R_1R_P}+\frac{q\,2N_1^2}{R_1\left(1+\frac{R_2+R_L}{R_T}\right)}+\frac{q\,4N_2^2}{R_P}+1\right)-\right.$$

$$\frac{i_A'}{s}\left(\frac{q^2N_1^2N_2N_3}{R_1\left(1+\frac{R_2+R_L}{R_T}\right)}+\frac{q^2N_1^3N_3}{R_1\left(1+\frac{R_T}{R_2+R_L}\right)}-qN_3(N_1+2N_2)\right)-$$

$$\left.\frac{i_B'}{s}qN_3(N_1-2N_2)+\frac{i_{3'}}{s}\left(\frac{q^2\,2N_1^2N_3^2}{R_P}+q\,2N_3^2\right)\right\}$$

if $R_T$ is approximately equal to $R_2+R_L$, that is if $R_1$ is much smaller than $R_T$, then $1+\frac{R_2+R_L}{R_T}$ is approximately 2

and, letting

$$a_0 = \frac{N_1^2}{R_P}, \quad a_1 = \frac{N_1^2}{R_1}, \quad a_2 = \frac{4N_2^2}{R_P}, \quad a_3 = \frac{2N_3^2}{R_3}$$

the direct component of $\bar{i}_3$ may be written

$$\frac{\frac{E_3}{R_3}\left[a_0a_1q^2+(a_1+a_2)q+1\right]+i_{3'}\left[a_0a_3q^2+a_3q\right]}{s\left[a_0(a_1+a_3)q^2+(a_1+a_2+a_3)q+1\right]}$$

where $i_{3'}$ is now the value of the direct component of $i_3$ existing at the end of the zone preceding. The denominator has the form

$$s\left[T_1T_2q^2+(T_1+T_2)q+1\right]$$

inwhich $T_1$ and $T_2$ are always real. This expression is valid for a step input ($E_3$) in signal voltage occurring at or before the initial zone boundary. The resulting transient will consist of two components of the L-R form having time constants $K\mu T_1$ and $K\mu T_2$ which may be obtained by use of the quadratic formula:

$$T \approx \frac{1}{\frac{1}{a_0} + \frac{4\alpha^2}{a_1+a_3} \pm \sqrt{\frac{1}{a_0^2} + \frac{8\alpha^2-4}{a_0(a_1+a_3)} + \frac{16\alpha^4}{(a_1+a_3)^2}}}$$

where $\alpha = \frac{N_2}{N_1}$. For the case $\alpha = \frac{1}{2}$,

$$T_1 = a_0 \qquad T_2 = a_1 + a_3$$

In Zone II, after saturation of core A

$$\bar{i}_3 = \frac{1}{R_3}\left\{\frac{1}{g\left(\frac{N_1^2}{R_1\left(1+\frac{R_2+R_L}{R_T}\right)} - \frac{2N_2(N_1-N_2)}{R_p} + \frac{N_3^2}{R_3}\right)+1}\right\}$$

$$\left\{\bar{e}_{1''}\, g\,\frac{N_3(N_1-2N_2)}{R_p} + \bar{e}_{3''}\left(g\frac{N_1^2}{R_1\left(1+\frac{R_2+R_L}{R_T}\right)} - g\,\frac{2N_2(N_1-N_2)}{R_p}+1\right)+\right.$$

$$\left.\frac{i_{A''}}{s}\, g N_2 N_3 - \frac{i_{B''}}{s}\, g N_3(N_1-N_2) + \frac{i_{3''}}{s}\, g N_3^2\right\}$$

For a step input, $E_3$, the denominator of the direct component takes the form

$$s(Tg+1)$$

if $R_1$ is much smaller than $R_T$,

$$T = \left(\frac{N_1^2}{2R_1} - \frac{2N_2(N_1-N_2)}{R_p} + \frac{N_3^2}{R_3}\right)$$

which is equal to one-half the value of $T_1 + T_2$ for Zone I, minus the term

$$\frac{2N_1N_2}{R_p}$$

The time constant of the resulting transient is $K\mu T$.

The D. C. component of $\bar{i}_3$ in Zone II is

$$\frac{\frac{E_3}{R_3}\left[(b_1-b_2)g+1\right]+i_{3''}\,b_3 g}{s\left[(b_1-b_2+b_3)g+1\right]} \qquad \text{where}$$

$$b_1=\frac{N_1^2}{2R_1}, \quad b_2=\frac{2N_2(N_1-N_2)}{R_P}, \quad b_3=\frac{N_3^2}{R_3}$$

and $i_{3''}$ is the direct component of $i_3$ at the end of Zone I.

The voltage equations given above are adequate only to the terminal boundary of Zone III. This boundary is determined by the time at which one of the components of load current changes sign, (Figure 4). For the case above considered, in which both components were initially positive, $i_B$ tends to go negative first. The voltage equations for the commutation zone may be set up by reversing the sense of $i_B$ in the feedback loop inboard of the rectifier bridge. Thus for Zone IV:

$$\bar{e}_{1IV}+Y_A=\left[g(N_1^2+2N_1N_2+2N_2^2)+R_T\right]\bar{i}_A+\left[2gN_2(N_2-N_1)-R_2+R_L\right]\bar{i}_B$$
$$+\left[gN_3(N_1+2N_2)\right]\bar{i}_3$$

$$\bar{e}_{1IV}+Y_B=\left[2gN_2^2+R_2+R_L\right]\bar{i}_A+\left[g(N_1^2+2N_2^2)+R_1-R_2+R_L\right]\bar{i}_B$$
$$-\left[gN_3(N_1-2N_2)\right]\bar{i}_3$$

$$\bar{e}_{3IV}+Y_3=\left[gN_3(N_1+2N_2)\right]\bar{i}_A-\left[gN_3(N_1+2N_2)\right]\bar{i}_B$$
$$+\left[2gN_3^2+R_3\right]\bar{i}_3$$

since the crossing of the initial and terminal boundaries of the commutating zone involves a rearrangement of the circuit rather than a change in

the parameters; the initial conditions, $y_A$, $y_B$ and $y_3$ , become more complicated than those heretofore considered. Since the object of this analysis is a description of the form of the direct component of signal current rather than a means to evaluate this component point by point, it will be necessary to determine boundary conditions only where a discontinuity in the d. c. component of the signal exists, namely at the initial and terminal boundaries of Zone II, in order to determine the form. Thus, at the boundaries of Zone IV, with linear parameters obtaining throughout the zone, and with the smooth switching effect inherent in the assumptions on the rectifiers, the direct component of signal current may be seen to be continuous. For simplicity the coefficients of the numerator of $\bar{i}_3$ will be indicated rather than determined, with the order of the numerator preserved, noting that $i_3$ on both sides of the initial boundary equals $i_{3^{IV}}$ .

Letting $R_{p'} = R_1 + 2R_L$ and

$$d_1 = \frac{N_1^2 + 4N_2^2}{R_{p'}} \quad , \quad d_2 = \frac{N_1(N_1 + 2N_2)}{R_1} \quad , \quad d_3 = \frac{2N_3^2}{R_3}$$

$$\bar{i}_3 = \frac{Fq^2 + Gq + 1}{S\left[d_1(d_2+d_3)q^2 + (d_1 + d_2 + d_3)q + 1\right]}$$

The denominator has the form

$$S\left[T_1 T_2 q^2 + (T_1 + T_2) q + 1\right]$$

in which $T_1 = d_1$ and $T_2 = d_2 + d_3$.

The resulting transient has two components with time constants $K\mu d_1$ and $K\mu(d_2 + d_3)$ respectively.

The argument for continuity of $i_3$ within this zone holds only if the step input $E_3$ is applied at an instant of time external to this zone. If, with the circuit in steady state, a step $E_3$ is applied within Zone IV, the direct component transform is

$$\bar{i}_3 = \frac{\frac{E_3}{R_3}\left[d_1 d_2 q^2 + (d_1 + d_2) q + 1\right]}{s\left[d_1 (d_2 + d_3) q^2 + (d_1 + d_2 + d_3) q + 1\right]}$$

and $i_3$ jumps to $\frac{E_3}{R_3} \frac{d_2}{(d_2 + d_3)}$ when the step input is applied.

The equations for $\bar{i}_3$ may be readily obtained for a step input $E_3$ occurring within the other zone boundaries. To determine the behavior of the direct component of signal current at the boundaries, it will be expedient to limit the expressions for $\bar{i}_3$ to include only such terms as affect the direct component, namely the terms in $\bar{e}_3$ and $i_{3'}$, $i_{3''}$, etc., defining $i_{3'}$, $i_{3''}$ etc. as the value of the direct component reached at the preceding boundary. Assuming the currents to be initially in the steady state such that $i_{3'}$ is constant and applying the step $E_3$ as the initial boundary of Zone I is crossed, the progress of $i_3$ across the succeeding intervals may be traced.

In Zone I, the term

$$\frac{i_{3'}\left[a_0 a_3 q^2 + a_3 q\right]}{s(T_1 q + 1)(T_2 q + 1)}$$

transforms to zero in the time equation since $i_{3'}$ has been assumed constant, thus

$$i_3 = \frac{E_3}{R_3}\left[1 - \frac{\left(\frac{a_0 a_1}{T_2} - (a_1 + a_2) + T_2\right)\epsilon^{-t/K\mu T_2}}{T_2 - T_1} + \frac{\left(\frac{a_0 a_1}{T_1} - (a_1 + a_2) + T_1\right)\epsilon^{-t/K\mu T_1}}{T_2 - T_1}\right]$$

there will be a jump in $i_3$ to the value $\frac{E_3}{R_3}\,\frac{a_0 a_1}{T_1 T_2}$ at the instant $E_3$ is applied. At the boundary of Zone II, $i_3$ has reached the value $i_3''$ and

$$\bar{i}_3 = \frac{\frac{E_3}{R_3}\left[(b_1 - b_2)q + 1\right] + i_3'' b_3 q}{s\left[(b_1 - b_2 + b_3)q + 1\right]}$$

another jump in $i_3$ will occur, to the value

$$\frac{\frac{E_3}{R_3}(b_1 - b_2) + i_3'' b_3}{b_1 - b_2 + b_3}$$

Again, as the boundary of Zone III is crossed, $i_3$ jumps to the value

$$\frac{\frac{E_3}{R_3} a_1 + i_3''' a_3}{a_1 + a_3}$$

By subtracting from the values acquired on crossing the boundaries, those which existed on reaching the boundaries, the magnitudes and directions of the jumps in the direct component of signal current at the limits of Zone II may be found:

| | Initial boundary | Terminal boundary |
|---|---|---|
| Magnitude of jump: | $\frac{\left(\frac{E_3}{R_3} - i_3''\right)(b_1 - b_2)}{b_1 - b_2 + b_3}$ | $\frac{\left(\frac{E_3}{R_3} - i_3'''\right) a_1}{a_1 + a_3}$ |
| Direction: | up (if $E_3$ is positive | ) up |

The change in the direct component of signal current may be described as a curve with discontinuities at the boundaries of Zone II and with varying slopes dependent upon the effective time constants in the separate zones. The pronounced jump occurring in the signal current when the step change in control voltage is applied has been noted and recorded on oscillograms by Beaumarriage et al. (3).

3. Series connection

A schematic diagram of the series connected, external feedback, magnetic amplifier is given in Figure 5. Since $i_A$ and $i_B$ are the same for this connection, only two voltage equations are required for Zones I, II, and III. These are developed in the same manner as for the parallel connection.

$$\bar{e}_1 + Y_1 = \left[ q_A (N_1+N_2)^2 + q_B (N_1-N_2)^2 + R_T \right] \bar{i}_1 + \left[ q_A N_3 (N_1+N_2) - q_B N_3 (N_1-N_2) \right] \bar{i}_3$$

$$\bar{e}_3 + Y_3 = \left[ q_A N_3 (N_1+N_2) - q_B N_3 (N_1-N_2) \right] \bar{i}_1 + \left[ q_A N_3^2 + q_B N_3^2 + R_3 \right] \bar{i}_3$$

where

$$Y_1 = \left[ q_A (N_1+N_2)^2 + q_B (N_1-N_2)^2 \right] \frac{i_1'}{s} + \left[ q_A N_3 (N_1+N_2) - q_B N_3 (N_1-N_2) \right] \frac{i_3'}{s}$$

$$Y_3 = \left[ q_A N_3 (N_1+N_2) - q_B N_3 (N_1-N_2) \right] \frac{i_1'}{s} + \left[ q_A N_3^2 + q_B N_3^2 \right] \frac{i_3'}{s}$$

$R_T = R_1 + R_2 + R_L$ $R_1$ is now the sum of the internal resistances of the main A and B windings.

In Zones I and III,

and

$$\bar{i}_3 = \left\{ \frac{1}{q^2 \left( \frac{2N_1^2}{R_T} \frac{2N_3^2}{R_3} \right) + q \left( \frac{2N_1^2}{R_T} + \frac{2N_2^2}{R_T} + \frac{2N_3^2}{R_3} \right) + 1} \right\}$$

$$\left\{ \frac{\bar{e}_3'}{R_3} \left( \frac{2N_1^2 + 2N_2^2}{R_T} q + 1 \right) - \frac{\bar{e}_1'}{R_3} \frac{2N_2 N_3}{R_T} q + \frac{i_1'}{s} \frac{2N_2 N_3}{R_3} q + \frac{i_3'}{s} \left( \frac{2N_1^2}{R_T} \frac{2N_3^2}{R_3} q^2 + \frac{2N_3^2}{R_3} q \right) \right\}$$

if we let $a_1 = \frac{2N_1^2}{R_T}$ , $a_2 = \frac{2N_2^2}{R_T}$ and $a_3 = \frac{2N_3^2}{R_3}$

the direct component of $\bar{i}_3$ becomes, for a step input $E_3$,

$$\frac{\frac{E_3}{R_3}\left[(a_1+a_2)q+1\right] + i_{3'}\left[a_1 a_3 q^2 + a_3 q\right]}{S\left[a_1 a_3 q^2 + (a_1+a_2+a_3)q + 1\right]}$$

The denominator has the form

$$s\left[T_1 T_2 q^2 + (T_1+T_2)q + 1\right]$$

in which $T_1$ and $T_2$ are again real and may be found by the quadratic formula.

In Zone II, $q_A = 0$ , $q_B = q$ and

$$\bar{i}_3 = \left\{\frac{1}{(b_1+b_3)q+1}\right\}\left\{\frac{\bar{e}_3}{R_3}(b_1 q+1) + \frac{\bar{e}_1}{R_3}\,\frac{N_3(N_1-N_2)}{R_T}q - \frac{i_{1''}}{S}\,\frac{N_3(N_1-N_2)}{R_T}q + i_{3''} b_3 q\right\}$$

in which $b_1 = \frac{(N_1-N_2)^2}{R_T}$ and $b_3 = \frac{N_3^2}{R_3}$

For a step input $E_3$, the direct component of signal current becomes

$$\frac{\frac{E_3}{R_3}(b_1 q+1) + i_{3''} b_3 q}{S\left[(b_1+b_3)q+1\right]}$$

As the commutation zone is entered, all the elements in the rectifier bridge become active and all the current in the feedback loop no longer goes through the load. To describe conditions in this zone, it will be necessary to adjust the voltage equations to account for that portion of the feedback current which does not flow through the load, but appears as a circulating component in the feedback loop. At the rectifier terminals,

$i_1$ flows in from the main windings

$i_1 + i_2$ flows in the feedback windings

$i_1$ flows from bridge to load resistor

Noting that $i_2$ flows across the rectifier bridge between the feedback terminals, the voltage equations may be simplified by observing that the drop through the feedback windings is zero in this zone. The adjusted voltage equations are:

$$\bar{e}_1 + y_1 = \left[g_A N_1(N_1+N_2) + g_B N_1(N_1-N_2) + R_1 + R_L\right]\bar{i}_1 + \left[g_A N_1N_2 - g_B N_1N_2\right]\bar{i}_2 + \left[g_A N_1N_3 - g_B N_1N_3\right]\bar{i}_3$$

$$0 + y_2 = \left[g_A N_2(N_1+N_2) - g_B N_2(N_1-N_2) + R_2\right]\bar{i}_1 + \left[g_A N_2^2 + g_B N_2^2 + R_2\right]\bar{i}_2 + \left[g_A N_2N_3 + g_B N_2N_3\right]\bar{i}_3$$

$$\bar{e}_3 + y_3 = \left[g_A N_3(N_1+N_2) - g_B N_3(N_1-N_2)\right]\bar{i}_1 + \left[g_A N_2N_3 + g_B N_2N_3\right]\bar{i}_2 + \left[g_A N_3^2 + g_B N_3^2 + R_3\right]\bar{i}_3$$

Throughout the zone $g_A = g_B = g$ and these equations reduce to:

$$\begin{aligned}
\bar{e}_1 + y_1 &= \left[2gN_1^2 + R_1 + R_L\right]\bar{i}_1 &&+ \quad 0 &&+ \quad 0 \\
0 + y_2 &= \left[2gN_2^2 + R_2\right]\bar{i}_1 &&+ \left[2gN_2^2 + R_2\right]\bar{i}_2 &&+ \left[2gN_2N_3\right]\bar{i}_3 \\
\bar{e}_3 + y_3 &= \left[2gN_2N_3\right]\bar{i}_1 &&+ \left[2gN_2N_3\right] i_2 &&+ \left[2gN_3^2 + R_3\right]\bar{i}_3
\end{aligned}$$

Realizing that the direct component of signal current will be continuous across the commutation zone, including the terminal instant of Zone III and the initial instant of the succeeding Zone I, as was discussed for the parallel connection, the form of the transient response in this

This jump is upward, the amount

$$\frac{\left( \frac{E_3}{R_3} - i_{3''} \right) b_1}{b_1 + b_3}$$

At the terminal boundary of Zone II, $i_3$ reaches the value $i_{3'''}$ .
On crossing this boundary,

$$i_3 = i_{3'''} \frac{a_1 a_3}{a_1 a_3}$$

that is, $i_3$ is continuous across the boundary. Within Zone II, the transient direct current level has but one component with time constant $K\mu(b_1 + b_3)$. On entering Zone III it resumes the two-component form which it had in Zone I. Then, in the commutating zone, the two components are modified, assuming the time constants

$$\frac{2K\mu N_1^2}{R_1 + R_L} \qquad \text{and} \qquad K\mu\left( \frac{2N_2^2}{R_2} + \frac{2N_3^2}{R_3} \right)$$

# CHAPTER III

# CONCLUSIONS

The futility of assigning a time constant to the transient change in the direct component of signal current occurring within a cycle of driving voltage has been demonstrated. Nevertheless, some conclusions may be drawn from the sectional characteristics developed in Chapter II.

The fact that larger time constants apply to the transients developed within the separate zones in the parallel connection than those in the series connection indicates that, with identical elements, the series connection should give more rapid response. However, the variations in direct signal current level due to the discontinuities encountered in the parallel arrangement on application of a step change in signal and on crossing the boundaries of zone II may be much greater than the sum of the changes occurring by continuous increase across the separate zones. This effect will rapidly diminish as the signal current approaches its ultimate d.c. level. The initial portion of transient response in the parallel circuit may thus be more rapid than that of the series circuit.

When the transient consists of two components of opposite sign, the component having the greater time constant has the larger magnitude, but the effect of the smaller component is a depression of the transient current at the beginning of the response, such that the resultant response is slower than that characterized by the larger time constant. This may be shown by considering the transform

$$\bar{i} = \frac{I\,(As^2 + Bs + 1)}{s\,[T_1T_2s^2 + (T_1 + T_2)s + 1]}$$

yielding the time function

$$i = I\left[1 - \frac{(A/T_2 - B + T_2)\,\epsilon^{-t/T_2}}{T_2 - T_1} + \frac{(A/T_1 - B + T_1)\,\epsilon^{-t/T_1}}{T_2 - T_1}\right]$$

in which $\left( \frac{A}{T_2} - B + T_2 \right) > 0$

and $\left( \frac{A}{T_1} - B + T_1 \right) > 0$

and $T_2 > T_1$

The component having time constant $T_2$ will be greater than that having time constant $T_1$ whenever

$$(T_2 - T_1)\left(1 - \frac{A}{T_1 T_2}\right) > 0$$

This condition is fulfilled wherever this form of transient is encountered in the parallel or series connection, as long as a transient exists.

It should be noted that any improvement in response characteristics achieved by variation of the $\frac{N^2}{R}$ ratios of the various windings will normally be accompanied by a decrease in power gain. The number of turns employed in the feedback winding may have only a minor effect on the response characteristics, but this number if limited by its effect on the stability of the amplifier. Excessive feedback produces a condition in which nearly full output is received at no signal and very low gain is obtained.

The difficulties which would be encountered in an extension of this method to a quantitative analysis are readily apparent. For this reason the results appear to be almost incapable of numerical verification. Experimental determination of the direct component of signal current is severely hampered by the relative magnitude of the alternating component of signal current with respect to the change in the direct component.

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## 1. Parallel connection.

$$\bar{e}_1 + Y_A = [q_A(N_1+N_2)^2 + q_B N_2^2 + R_T]\bar{i}_A + [q_A N_2(N_1+N_2) + q_B N_2(N_2-N_1) + R_2 + R_L]\bar{i}_B$$
$$+ [q_A N_3(N_1+N_2) + q_B N_2 N_3]\bar{i}_3$$

$$\bar{e}_1 + Y_B = [q_A N_2(N_1+N_2) + q_B N_2(N_2-N_1) + R_2 + R_L]\bar{i}_A + [q_A N_2^2 + q_B(N_1-N_2)^2 + R_T]\bar{i}_B$$
$$+ [q_A N_2 N_3 + q_B N_3(N_2-N_1)]\bar{i}_3$$

$$\bar{e}_3 + Y_3 = [q_A N_3(N_1+N_2) + q_B N_2 N_3]\bar{i}_A + [q_A N_2 N_3 + q_B N_3(N_2-N_1)]\bar{i}_B$$
$$+ [q_A N_3^2 + q_B N_3^2 + R_3]\bar{i}_3$$

where : $R_T = R_1 + R_2 + R_L$ and

$Y_A, Y_B, Y_3$ are terms in initial conditions

In zones I and III $q_A = q_B = q$ and $e_1 = E_1 \sin(\omega t + \theta_I)$ or $(\cdots \theta_{III})$

$$\bar{e}_1 + Y_A = [q(N_1^2 + 2N_1N_2 + 2N_2^2) + R_T]\bar{i}_A + [2qN_2^2 + R_2 + R_L]\bar{i}_B$$
$$+ [q N_3(N_1 + 2N_2)]\bar{i}_3$$

$$\bar{e}_1 + Y_B = [2qN_2^2 + R_2 + R_L]\bar{i}_A + [q(N_1^2 - 2N_1N_2 + 2N_2^2) + R_T]\bar{i}_B$$
$$- [q N_3(N_1 - 2N_2)]\bar{i}_3$$

$$\bar{e}_3 + Y_3 = [q N_3(N_1 + 2N_2)]\bar{i}_A - [q N_3(N_1 - 2N_2)]\bar{i}_B$$
$$+ [2qN_3^2 + R_3]\bar{i}_3$$

# APPENDIX

## 1. Parallel connection.

Initial conditions not specifically determined for zone I.

For zone III:

$$Y_A = g\,(N_1^2 + 2N_1N_2 + 2N_2^2)\, i_{A'''}/S + 2g N_2^2\, i_{B'''}/S + g N_3 (N_1 + 2N_2)\, i_{3'''}/S$$

$$Y_B = 2g N_2^2\, i_{A'''}/S + g\,(N_1^2 - 2N_1N_2 + 2N_2^2)\, i_{B'''}/S - g N_3 (N_1 - 2N_2)\, i_{3'''}/S$$

$$Y_3 = g N_3 (N_1 + 2N_2)\, i_{A'''}/S - g N_3 (N_1 - 2N_2)\, i_{B'''}/S + 2g N_3^2\, i_{3'''}/S$$

$$\bar{i}_3 = \frac{-\bar{e}_1\, 4g N_2 N_3 R_1 + \bar{e}_3 \left[ g^2 N_1^4 + g\,(N_1^2 + 4N_2^2) R_1 + g N_1^2 R_p + R_1 R_p \right]}{g^2 \left[ N_1^2 (N_1^2 R_3 + 2N_3^2 R_1) \right] + g \left[ (N_1^2 + 4N_2^2) R_3 R_1 + N_1^2 R_3 R_p + 2N_3^2 R_1 R_p \right] + R_1 R_3 R_p}$$

$$+ \frac{i_{A'''} \left[ g N_3 (N_1 + 2N_2) R_1 R_p - g^2 N_1^2 (N_2 N_3 R_T + N_1 N_3 R_2 + N_1 N_3 R_L) \right]}{S \left\{ g^2 \left[ N_1^2 (N_1^2 R_3 + 2N_3^2 R_1) \right] + g \left[ (N_1^2 + 4N_2^2) R_3 R_1 + N_1^2 R_3 R_p + 2N_3^2 R_1 R_p \right] + R_1 R_3 R_p \right\}}$$

$$+ \frac{-i_{B'''} \left[ g N_3 (N_1 - 2N_2) R_1 R_p \right] + i_{3'''} \left[ 2g N_1^2 N_3^2 R_1 + 2g N_3^2 R_1 R_p \right]}{S \left\{ g^2 \left[ N_1^2 (N_1^2 R_3 + 2N_3^2 R_1) \right] + g \left[ (N_1^2 + 4N_2^2) R_3 R_1 + N_1^2 R_3 R_p + 2N_3^2 R_1 R_p \right] + R_1 R_3 R_p \right\}}$$

Where $R_p = R_1 + 2R_2 + 2R_L$

If $\frac{\bar{e}_3}{R_3} = \frac{E_3}{R_3 S}$, the direct component transform is

$$\bar{i}_3 = \frac{\frac{E_3}{R_3} \left[ g^2 \frac{N_1^4}{R_1 R_p} + g \frac{2N_1^2}{R_1 \left(1 + \frac{R_2 + R_L}{R_T}\right)} + g \frac{4N_2^2}{R_p} + 1 \right] + i_{3'''} \left[ g^2 \frac{2N_1^2 N_3^2}{R_p R_3} + g \frac{2N_3^2}{R_3} \right)}{S \left\{ g^2 \left[ \frac{N_1^2}{R_p} \left( \frac{N_1^2}{R_1} + \frac{2N_3^2}{R_3} \right) \right] + g \left[ \frac{2N_1^2}{R_1 \left(1 + \frac{R_2 + R_L}{R_T}\right)} + \frac{4N_2^2}{R_p} + \frac{2N_3^2}{R_3} \right] + 1 \right\}}$$

1. Parallel connection.

$\bar{e}_1$ at initial boundary of zone II is $\overline{E_1 \sin(\omega t + \theta_{II})}$ so that

$\bar{e}_1$ for zone II $\neq$ $\bar{e}_1$ for other zones.

For zone II :

$$\bar{e}_1 + Y_A = \left[g N_2^2 + R_T\right]\bar{i}_A + \left[g N_2(N_2 - N_1) + R_2 + R_L\right]\bar{i}_B + \left[g N_2 N_3\right]\bar{i}_3$$

$$\bar{e}_1 + Y_B = \left[g N_2(N_2 - N_1) + R_2 + R_L\right]\bar{i}_A + \left[g (N_1 - N_2)^2 + R_T\right]\bar{i}_B + \left[g N_3(N_2 - N_1)\right]\bar{i}_3$$

$$\bar{e}_3 + Y_3 = \left[g N_2 N_3\right]\bar{i}_A + \left[g N_3(N_2 - N_1)\right]\bar{i}_B + \left[g N_3^2 + R_3\right]\bar{i}_3$$

$$Y_A = g N_2^2\, i_{A''}/S + g N_2(N_2 - N_1)\, i_{B''}/S + g N_2 N_3\, i_{3''}/S$$

$$Y_B = g N_2(N_2 - N_1)\, i_{A''}/S + g (N_1 - N_2)^2\, i_{B''}/S + g N_3(N_2 - N_1)\, i_{3''}/S$$

$$Y_3 = g N_2 N_3\, i_{A''}/S + g N_3(N_2 - N_1)\, i_{B''}/S + g N_3^2\, i_{3''}/S$$

$$\bar{i}_3 = \frac{\bar{e}_1\left[g N_3(N_1 - 2N_2)R_1\right] + \bar{e}_3\left[g N_1^2 R_T - 2 g N_2(N_1 - N_2)R_1 + R_1 R_P\right]}{g N_1^2 R_3 R_T - 2 g N_2(N_1 - N_2) R_1 R_3 + g N_3^2 R_1 R_P + R_1 R_3 R_P}$$

$$+ \frac{i_{A''}\left[g N_2 N_3 R_1 R_P\right] - i_{B''}\left[g N_3(N_1 - N_2) R_1 R_P\right] + i_{3''}\left[g N_3^2 R_1 R_P\right]}{S\left\{g\left[N_1^2 R_3 R_T - 2 N_2(N_1 - N_2) R_1 R_3 + N_3^2 R_1 R_P\right] + R_1 R_3 R_P\right\}}$$

If $e_3 = E_3$, the direct component transform is

$$\bar{i}_3 = \frac{\frac{E_3}{R_3}\left[g \frac{N_1^2}{R_1\left(1 + \frac{R_2 + R_L}{R_T}\right)} - g\frac{2N_2(N_1 - N_2)}{R_P} + 1\right] + i_{3''}\left[g\ \frac{N_3^2}{R_3}\right]}{S\left\{g\left[\frac{N_1^2}{R_1\left(1 + \frac{R_2 + R_L}{R_T}\right)} - \frac{2N_2(N_1 - N_2)}{R_P} + \frac{N_3^2}{R_3}\right] + 1\right\}}$$

# APPENDIX

## 1. Parallel connection.

In commutation zone, $q_A = q_B = q$ and sense of $i_B$ is reversed in feed back loop.

$$\bar{e}_1 + y_A = \left[ q \left( N_1^2 + 2N_1N_2 + 2N_2^2 \right) + R_T \right] \bar{i}_A + \left[ 2q N_2 (N_2 - N_1) - R_2 + R_L \right] \bar{i}_B$$
$$+ \left[ q N_3 (N_1 + 2N_2) \right] \bar{i}_3$$

$$\bar{e}_1 + y_B = \left[ 2q N_2^2 + R_2 + R_L \right] \bar{i}_A + \left[ q (N_1^2 + 2N_2^2) + R_1 - R_2 + R_L \right] \bar{i}_B$$
$$- \left[ q N_3 (N_1 - 2N_2) \right] \bar{i}_3$$

$$\bar{e}_3 + Y_3 = \left[ q N_3 (N_1 + 2N_2) \right] \bar{i}_A - \left[ q N_3 (N_1 + 2N^2) \right] \bar{i}_B + \left[ 2q N_3^2 + R_3 \right] \bar{i}_3$$

INITIAL CONDITIONS NOT SPECIFIED FOR ZONE IV

$$\bar{i}_3 = \frac{\left[ Y_B - Y_A \right] q N_3 \left[ N_1 + 2N_2 \right] \left[ q (N_1^2 + 4N_2^2) + R_1 + 2R_L \right]}{\begin{array}{l} q^2 \left[ N_1^2 + 2N_1N_2 \right] \left[ N_1^2 + 4N_2^2 \right] R_3 + q^2 \left[ 2N_3^2 \right] \left[ N_1^2 + 4N_2^2 \right] R_1 + \\ q \left[ (N_1^2 + 4N_2^2) R_1 R_3 + (N_1^2 + 2N_1N_2) R_3 (R_1 + 2R_L) + 2N_3^2 R_1 (R_1 + 2R_L) \right] + \\ R_1 R_3 (R_1 + 2R_L) \end{array}}$$

$$+ \frac{\bar{e}_3 \left\{ q^2 \left[ N_1^2 + 2N_1N_2 \right] \left[ N_1^2 + 4N_2^2 \right] + q \left[ (N_1^2 + 4N_2^2) R_1 + (N_1^2 + 2N_1N_2)(R_1 + 2R_L) \right] + R_1 (R_1 + 2R_L) \right\}}{\text{same denominator}}$$

Form of direct component transform:

$$\bar{i}_3 = \frac{F s^2 + G s + 1}{s \left\{ q^2 \left[ \frac{(N_1^2 + 4N_2^2)(N_1^2 + 2N_1N_2)}{R_1 (R_1 + 2R_L)} + \frac{2N_3^2 (N_1^2 + 4N_2^2)}{R_3 (R_1 + 2R_L)} \right] + q \left[ \frac{N_1^2 + 4N_2^2}{R_1 + 2R_L} + \frac{N_1^2 + 2N_1N_2}{R_1} + \frac{2N_3^2}{R_3} \right] + 1 \right\}}$$

2. Series connection.

For zones I, II and III

$$\bar{e}_1 + Y_1 = \left[g_A (N_1+N_2)^2 + g_B (N_1-N_2)^2 + R_T\right]\bar{i}_1 + \left[g_A N_3 (N_1+N_2) - g_B N_3 (N_1-N_2)\right]\bar{i}_3$$

$$\bar{e}_3 + Y_3 = \left[g_A N_3 (N_1+N_2) - g_B N_3 (N_1-N_2)\right]\bar{i}_1 + \left[g_A N_3^2 + g_B N_3^2 + R_3\right]\bar{i}_3$$

For zone I, $e_1 = E_1 \sin(\omega t + \theta_1)$, *initial conditions not specified.*

$$g_A = g_B = g$$

$$\bar{i}_3 = \frac{\left[\bar{e}_3 + Y_3\right]\left[2g(N_1^2+N_2^2) + R_T\right] - \left[\bar{e}_1 + Y_1\right]\left[2g N_2 N_3\right]}{4g^2 N_1^2 N_3^2 + 2g\left[(N_1^2+N_2^2)R_3 + N_3^2 R_T\right] + R_3 R_T}$$

If $\bar{e}_3 = E_3/s$, *the direct component transform becomes*

$$\bar{i}_3 = \frac{\left[\frac{E_3}{R_3} + \frac{Y_3}{R_3}s\right]\left[g\frac{2N_1^2 + 2N_2^2}{R_T} + 1\right] - \left[\frac{Y_1}{R_3}s\right]\left[2g N_2 N_3\right]}{s\left\{g^2\left[\frac{2N_1^2}{R_T}\,\frac{2N_3^2}{R_3}\right] + g\left[\frac{2N_1^2+2N_2^2}{R_T} + \frac{2N_3^2}{R_3}\right] + 1\right\}}$$

with terms in $Y_1$ and $Y_3$ due to *alternating* currents suppressed.

For zone III, $Y_3 = 2g N_2 N_3\, i_{1'''}/s + 2g N_3^2\, i_{3'''}/s$

$$Y_1 = 2g(N_1^2 + N_2^2)\, i_{1'''}/s + 2g N_2 N_3\, i_{3'''}/s$$

and direct component transform is

$$\bar{i}_3 = \frac{\frac{E_3}{R_3}\left[g\left(\frac{2N_1^2 + 2N_2^2}{R_T}\right) + 1\right] + i_{3'''}\left[g^2\left(\frac{2N_1^2}{R_T}\,\frac{2N_3^2}{R_3}\right) + g\left(\frac{2N_3^2}{R_3}\right)\right]}{s\left\{g^2\left[\frac{2N_1^2}{R_T}\,\frac{2N_3^2}{R_3}\right] + g\left[\frac{2N_1^2+2N_2^2}{R_T} + \frac{2N_3^2}{R_3}\right] + 1\right\}}$$

2. Series connection.

In zone II , $q_A = 0$ , $q_B = q$ , $e_1 = E_1 \sin(\omega t + \theta_{11})$ , $e_3 = E_3$

$$\bar{e}_1 + Y_1 = [q(N_1 - N_2)^2 + R_T]\bar{i}_1 - [qN_3(N_1 - N_2)]\bar{i}_3$$

$$\bar{e}_3 + Y_3 = -[qN_3(N_1 - N_2)]\bar{i}_1 + [qN_3^2 + R_3]\bar{i}_3$$

$$Y_1 = q(N_1 - N_2)^2 i_{1''}/s - qN_3(N_1 - N_2)i_{3''}/s$$

$$Y_3 = -qN_3(N_1 - N_2)i_{1''}/s + qN_3^2 i_{3''}/s$$

$$\bar{i}_3 = \frac{[\bar{e}_3 + Y_3][q(N_1 - N_2)^2 + R_T] + [\bar{e}_1 + Y_1][qN_3(N_1 - N_2)]}{q[(N_1 - N_2)^2 R_3 + N_3^2 R_T] + R_3 R_T}$$

and the direct component transform is

$$\bar{i}_3 = \frac{\frac{E_3}{R_3}\left[q\frac{(N_1 - N_2)^2}{R_T} + 1\right] + i_{3''}\left[q\ \frac{N_3^2}{R_3}\right]}{s\left\{q\left[\frac{(N_1^2 - N_2)^2}{R_T} + \frac{N_3^2}{R_3}\right] + 1\right\}}$$

Commutation is reached when the voltage drop through the feedback loop becomes zero. Thus, in zone IV, the feedback current is no longer equal to $i_1$, but may differ from $i_1$, and will be called $i_2$ in the voltage equations for this zone. $q_A = q_B = q$

$$\bar{e}_1 + Y_1 = [q_A N_1(N_1 + N_2) + q_B N_1(N_1 - N_2) + R_1 + R_L]\bar{i}_1 + [q_A N_1 N_2 - q_B N_1 N_2]\bar{i}_2 + [q_A N_1 N_3 - q_B N_1 N_3]\bar{i}_3$$

$$Y_2 = [q_A N_2(N_1 + N_2) - q_B N_2(N_1 - N_2) + R_2]\bar{i}_1 + [q_A N_2^2 + q_B N_2^2 + R_2]\bar{i}_2 + [q_A N_2 N_3 + q_B N_2 N_3]\bar{i}_3$$

$$\bar{e}_3 + Y_3 = [q_A N_3(N_1 + N_2) - q_B N_3(N_1 - N_2)]\bar{i}_1 + [q_A N_2 N_3 + q_B N_2 N_3]\bar{i}_2 + [q_A N_3^2 + q_B N_3^2 + R_3]\bar{i}_3$$

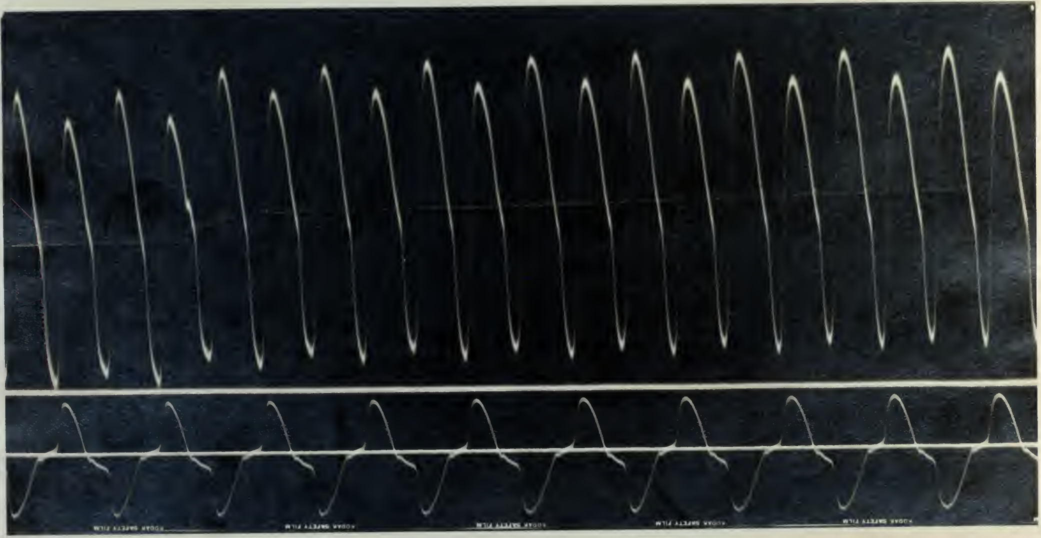
Initial conditions not specified.

2. Series connection.

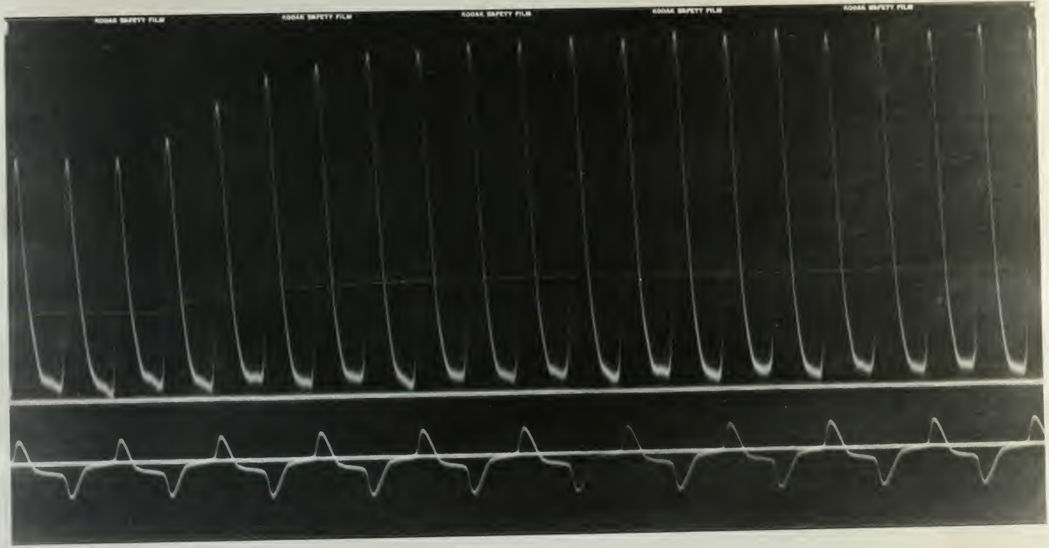
$$\bar{i}_3 = \frac{[\bar{e}_3 + Y_3][2 g N_2^2 + R_2] - Y_2 [2 g N_2 N_3]}{g[2 N_2^2 R_3 + 2 N_3^2 R_2] + R_2 R_3}$$

If $e_3 = E_3$ ,

$$\bar{i}_3 = \frac{\left[\frac{E_3}{R_3} + \frac{Y_3}{R_3} s\right]\left[g \frac{2 N_2^2}{R_2} + 1\right] - \left[\frac{Y_2}{R_3} s\right]\left[g \frac{2 N_2 N_3}{R_2}\right]}{s \left\{ g \left[\frac{2 N_2^2}{R_2} + \frac{2 N_3^2}{R_3}\right] + 1 \right\}}$$



Parallel connection. Response to step input $E_z$ . Top: signal current, bottom: load current (scales differ). Average value of $i_z$ shown by yellow line.



Series connection. Response to step input $E_z$ . Top: signal current, bottom: load current (scales differ). Average value of $i_z$ shown by yellow line.

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